permutations

Formulae:-

-> Factorial Notation :- Let n be positive integer.Then ,factorial n
        dentoed by n! is defined as
$$n! = n(n-1)(n-2). . . . . . . .3.2.1$$
            eg:- 5! = (5 * 4* 3 * 2 * 1)
                        = 120
                    0! = 1

->Permutations :- The different arrangements of a given number of things
    by taking some or all at a time,are called permutations.
            eg:- All permutations(or arrangements)made with the letters
a,b,c by
                taking two at a time are (ab,ba,ac,ca,bc,cb)

->Numbers of permutations :- Number of all permutations of n things ,
        taken r at a time is given by
$$nPr = n(n-1)(n-2). . . .. . . (n-r+1)$$
$$= n! / (n-r)!$$

->An Important Result :- If there are n objects of which p1 are alike of
one
        kind ; p2 are alike of another kind ; p3 are alike of third kind and
so on
        and pr are alike of rth kind, such that (p1+p2+. . . . . pr) =
n
Then,number of permutations of these n objects is:
$$n! / (p1!).(p2!). . . . .(pr!)$$

->Combinations :- Each of different groups or selections which can be
    formed by taking some or all of a number of objects,is called a
    combination.
                eg:- Suppose we want to select two out of three boys A,B,C
.
                    then ,possible selection are AB,BC & CA.
        Note that AB and BA represent the same selection.

-> Number of Combination :- The number of all combination of n things
        taken r at atime is:

$$nCr = n! / (r!)(n-r)!$$
$$= n(n-1)(n-2). . . . . . . tor factors /$$
r!

Note that : nCn = 1 and nC0 =1

An Important Result : nCr = nC(n-r)

PROBLEMS

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1.Evaluate 30!/28!
Sol:-          30!/28! = 30 * 29 * (28!)  / (28!)
                                = 30 * 29 =870



2.Find the value of 60P3
Sol:- 60P3  = 60! / (60 – 3)! = 60! / 57!
                                    = (60 * 59 *58 * (57!) )/ 57!
                                    = 60 * 59 *58
                                     = 205320



3. Find the value of 100C98        50C 50
Sol:-             100C98  = 100C100-98)
                         = 100 * 99 / 2 *1
                         = 4950
                    50C50 = 1


4.How many words can be formed by using all the letters of the word
"DAUGHTR"⁷₈ so that vowels always come together & vowels are never
together?
Sol:- (i) Given word contains 8 different letters
              When the vowels AUE are always together we may suppose
              them to form an entity ,treated as one letter
              then the letter to be arranged are DAHTR(AUE)
              these 6 letters can be arranged in 6p6 = 6!
                                          = 720 ways
          The vowels in the group (AUE) may be arranged in 3! = 6 ways
          Required number of words = 760 * 6 =4320


(ii)Total number of words formed by using all the letters of the given
   words
              8! = 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1
                      = 40320
          Number of words each having vowels together
                 = 760 * 6
                 = 4320
          Number of words each having vowels never together
                 = 40320 – 4320
                 = 36000



5.In how many ways can a cricket eleven be chosen out of a batch
of 15 players.
Sol:- Required number of ways
                          = 15C 11 = 15C (15-11)
                          = 15 C 4
                 15C4 = 15 * 14 * 13 * 12 / 4 * 3 * 2 *1
                      = 1365

6.In how many  a committee of 5 members can be selected from 6men
5 ladies consisting of 3 men and 2 ladies
Sol:-          (3 men out of 6) and (2 ladies out of 5) are to be chosen
              Required number of ways
                              =(6C3 * 5C2)
                              = 200



7.How many 4-letter word with or without meaning can be formed out
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of the letters of the word 'LOGARITHMS' if repetition of letters is
not allowed
Sol:-   'LOGARITHMS' contains 10 different letters
                  Required number of words
                        = Number of arrangements of 100 letters taking
                              4 at a time
                        = 10P4
                        = 10 * 9 * 8 * 7
                        = 5040

8.In how many ways can the letter of word 'LEADER' be arranged
Sol:-   The word 'LEADER' contains 6 letters namely
                        1L,2E,1A,1D and 1R
                  Required number of ways
                        = 6! / (1!)(2!)(1!)(1!)(1!)
                        = 6 * 5 * 4 * 3 * 2 *1 / 2 * 1
                        =360

9.How many arrangements can be made out of the letters of the word
'MATHEMATICS' be arranged so that the vowels always come
together
Sol:-   In the word ' MATHEMATICS' we treat vowels
            AEAI as one letter thus we have MTHMTCS(AEAI)
            now we have to arrange 8 letters out of which M occurs
            twice ,T occurs twice & the rest are different
         Number of ways of arranging these letters
                        = 8! / (2!)(2!)
                        = 10080
now AEAI has 4 letters in which A occurs 2 times and the rest
are different
         Number of ways of arranging these letters
                        = 4! / 2! = 12
         Required number of words = (10080 * 12)
                              = 120960

10.In how many different ways can the letter of the word 'DETAIL' be
arranged in such a way that the vowels occupy only the odd positions
Sol:-   These are 6 letters in the given word , out of which
            there are 3 vowels and 3 consonants
         Let us mark these positions as under

                        (1)(2) (3) (4)(5)(6)
            now 3 vowels can be placed at any of the three places out of 4
               marked 1,3,5
      Number of ways of arranging the vowels = 3P3 = 3! =6
       Also,the 3 consonants can be arranged at the remaining 3 positions
      Number of arrangements = 3P3 = 6
       Total number of ways = (6 * 6) =36

11.How many 3 digit numbers can be formed from the digits 2,3,5,6,7
 and 9 which are divisible by 5 and none of the digits is repeated?
Sol:- Since each desired number is divisible by 5,
         so we much have 5 at the unit place.   The hundreds place
         can now be filled by any of the remaining 4 digits .so, there
         4 ways of filling it.
         Required number of numbers = (1 * 5 * 4)

12.In how many ways can 21 books on English and 19 books on Hindi
 be placed in a row on a self so that two books on Hindi may not
 be together?
Sol:-   In order that two books on Hindi are never together,
        we must place all these books as under:
                        X E X E X . . . . X E X
        Where E denotes the position of an English and X that of
         a Hindi book.
        Since there are 21 books on English,the number of places
        marked X are therefore 22.
        Now, 19 places out of 22 can be chosen in
                    22 C 19 = 22 C 3 =22 * 21 * 20 / 3 * 2 *1
        Hence the required number of ways = 1540

13.Out of 7 constants and 4 vowels how many words of 3 consonants
   and 2 vowels can be formed?
Sol:-   Number of ways of selecting (3 consonants out of 7) and
                                        (2 vowels out of 4)

                        = 7C3 * 4C2
                         = 210
            Number of groups each having 3 consonants and 2 vowels = 210
            Each group contains 5 letters
            Number of ways of arranging 5 letters among themselves
                    = 5! = (5 * 4 * 3 * 2 * 1)
                         = 210
          Required number of words = (210 * 210)
                              = 25200